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Decomposition-Based Exact Algorithms for Two-Stage Flexible Flow Shop Scheduling with Unrelated Parallel Machines

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Agenda

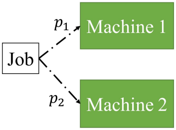
1. Introduction
2. MIP Model
3. Decomposition
4. Tuning
5. Computational Results
6. Conclusion

Problem Definition

Flexible Flow Shop Scheduling Problem

Flow Shop: Given a set of *jobs* to be processed on a set of *stages* following **the same route**.

Flexible: Each stage can have a single or multiple *parallel machines*



1. **Identical**: $p_1 = p_2$
2. **Uniform**: $p_1 = \alpha * p_2$
   where α is machine-speed factor
3. **Unrelated**: $p_1 \neq p_2$

Goal: Find the optimal job schedule with respect to a certain objective value

Example of Two-Stage Flexible Flow Shop

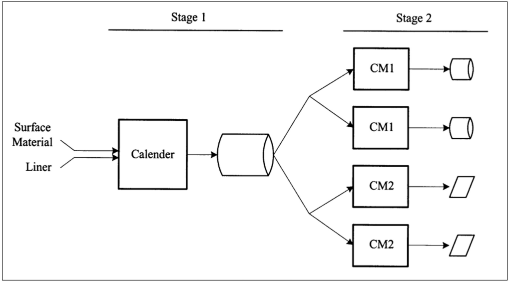


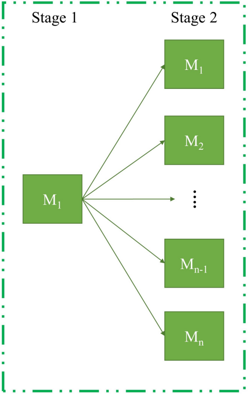
Figure: Two Stages Manufacturing System. (Lin and Liao 2003)

Objectives

1. We study two-stage flexible flow shop problem with unrelated parallel machines, i.e., $\boldsymbol{FF2|(1, RM)|C_{max}}$

   $FF2$: two-stage FFSP
   $(1, RM)$: single machine in stage 1
   unrelated parallel machines in stage 2
   C_{max}: makespan minimization

2. To the best of our knowledge, this is the first study to implement decomposition-based algorithms for solving flexible flow shop problem
   - Logic-Based Benders Decomposition
   - Branch-and-Check



Best known Mixed-Integer Programming Model

Follow the literature (Demir and İşleyen 2013), we develop a disjunctive MIP model for $FF2|(1, RM)|C_{max}$

Index

- $i \in \mathcal{I}$, index for machines
- $j \in \mathcal{J}$, index for jobs
- $k \in \mathcal{K}$, index for stages

Decision Variables

- $C_{max} \geq 0$, Makespan
- S_{kij}, $C_{kij} \geq 0$, Starting and completion time of job j on machine i in stage k
- $V_{kij} \in \{0, 1\}$, job-machine assignment
  E.g., $V_{kij} = 1$ if job j is assigned to machine i in stage k
- $X_{kijg} \in \{0, 1\}$, job sequence variables
  E.g., $X_{kijg} = 1$ if job j precedes job g on machine i in stage k

Data/Input

- p_{kij}, process time of job j on machine i in stage k

$$
\begin{aligned}
\text{Minimize} \quad & C_{max} && && (1)\\
\text{subject to} \quad & \sum_{i \in \mathcal{I}^{(2)}} V_{2ij} = 1 && j \in \mathcal{J} && (2)\\
& C_{max} \geq \sum i \in \mathcal{I} C_{2ij} && j \in \mathcal{J} && (3)\\
& S_{kij} + C_{kij} \leq V_{kij} M && j \in \mathcal{J}, i \in \mathcal{I}^{(2)}, k \in \mathcal{K} && (4)\\
& C_{kij} - p_{kij} \geq S_{kij} - (1 - V_{kij}) M && j \in \mathcal{J}, i \in \mathcal{I}^{(2)}, k \in \mathcal{K} && (5)\\
& S_{kij} \geq C_{kig} - (X_{kijg}) M && j, g \in \mathcal{J}, i \in \mathcal{I}^{(k)}, k \in \mathcal{K} && (6)\\
& S_{kig} \geq C_{kij} - (1 - X_{kijg}) M && j, g \in \mathcal{J}, i \in \mathcal{I}^{(k)}, k \in \mathcal{K} && (7)\\
& \sum_{i \in \mathcal{I}^{(2)}} S_{2ij} \geq \sum_{i \in \mathcal{I}^{(1)}} C_{1ij} && j \in \mathcal{J} && (8)\\
& S_{kij}, C_{kij} \geq 0; V_{kij}, X_{kijg} \in \{0, 1\} && j, g \in \mathcal{J}, i \in \mathcal{I}^{(k)}, k \in \mathcal{K} && (9)
\end{aligned}
$$

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\begin{aligned}
\text{Minimize} \quad & C_{max} && && (1)\\
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\end{aligned}
$$

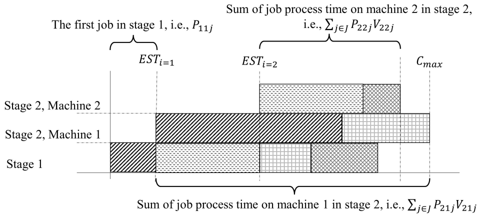
Additional Constraints

Job-Machine Assignment in Stage 1:

$$\sum_{j \in \mathcal{J}} V_{1ij} = 0, \; i = \{2, ..., n\} \quad (10)$$

Lower Bound Constraint:

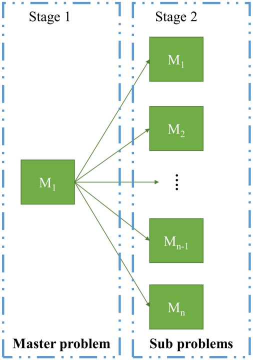
$$C_{max} \geq \min_{j \in \mathcal{J}} p_{11j} + \sum_{j \in \mathcal{J}} p_{2ij} V_{2ij}, \; i \in \mathcal{I}^{(2)} \quad (11)$$



Decomposition-Based Exact Algorithms $FF2|(1, RM)|C_{max}$

Master Problem: job sequencing in stage 1
job-machine assignment in stage 2

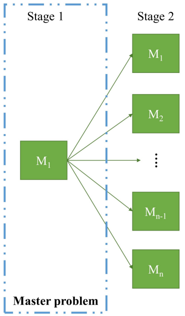
Sub-problem: job sequencing in stage 2



Mixed-Integer Programming Master Problems

Master Problem: relaxation of job-sequence on machines in stage 2.
It provides lower bound value $\underline{Z}$.

Minimize C_{max}		(12)
Subject to		
Const. (2) - (5), (8), (10), (11)		
$S_{1ij} \geq C_{1ig} - (X_{1ijg})M$	$j, g \in \mathcal{J}, i \in \mathcal{I}^{(1)},$	(13)
$S_{1ig} \geq C_{1ij} - (1 - X_{1ijg})M$	$j, g \in \mathcal{J}, i \in \mathcal{I}^{(1)}$	(14)
Benders cuts		(15)
$S_{kij}, C_{kij} \geq 0; V_{kij}, X_{kijg} \in \{0, 1\}$	$j, g \in \mathcal{J}, i \in \mathcal{I}, k \in \mathcal{K}$	(16)



Constraint Programming Sub-Problems

Sub-Problems: job-sequence on machine $i \in \mathcal{I}$ in stage 2.
It provides upper bound value $\overline{Z}$.

Decision Variables:
Interval Variables $\boldsymbol{job_j} = \{\boldsymbol{start}, \boldsymbol{end}, \boldsymbol{duration}\} \quad j \in \mathcal{J}$:

$$\text{Minimize} \quad C^{ih}_{max} \tag{17}$$

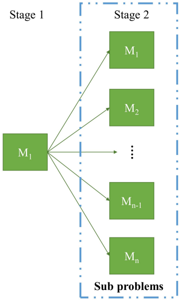
Subject to

$$C^{ih}_{max} \geq job_j.end \qquad j \in \{\mathcal{J} \mid \boldsymbol{V_{2ij} = 1}\} \tag{18}$$

$$job_j.duration = p_{kij} \qquad j \in \{\mathcal{J} \mid \boldsymbol{V_{2ij} = 1}\} \tag{19}$$

$$job_j.start \geq C_{11j} \qquad j \in \{\mathcal{J} \mid \boldsymbol{V_{2ij} = 1}\} \tag{20}$$

$$\boldsymbol{NoOverlap(job_j)} \qquad j \in \{\mathcal{J} \mid \boldsymbol{V_{2ij} = 1}\} \tag{21}$$



Benders Cut & Optimality Conditions

Benders Optimality Cuts in *iteration h*:

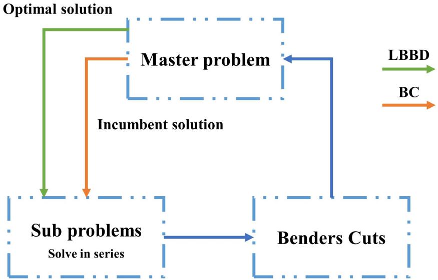
- Remove current solution in future
- Do not remove optimal solutions

$$C_{max} \geq \underbrace{Z_{sp}^h}_{C_{max}\text{ from SP in iteration h}} \underbrace{\underbrace{(1 - \sum_{i \in \mathcal{I}^{(2)}, j \in \mathcal{J} : \hat{V}_{2ij}^h = 1} (1 - V_{2ij})}_{\text{Stage 2: job assignment}} - \underbrace{\sum_{j,g \in \mathcal{J} : \hat{X}_{11jg}^h = 1} (1 - X_{11jg}))}_{\text{Stage 1: job sequencing}}}_{\text{Solution from MP in iteration h}}$$

Optimality Conditions: $\underline{Z} \leq Z^* \leq \overline{Z}$

Two Different Approaches

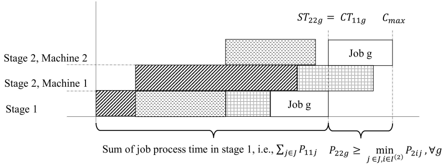
- **Logic-Based Bender Decomposition** (LBBD) (Hooker 2005; Tran, Araujo, and Beck 2016)
- **Branch-and-Check** (BC) (Thorsteinsson 2001; Beck 2010)



Tuning

Heuristic lower bound

$$C_{max} \geq \sum_{j \in \mathcal{J}} p_{11j} + \min_{j \in \mathcal{J}, i \in \mathcal{I}^{(2)}} p_{2ij}$$



Tightening Big-M

$$V_{2ij} * M \geq S_{2ij} + C_{2ij} \quad j \in \mathcal{J}, i \in \mathcal{I}^{(2)} \quad (4)$$

$$\Downarrow$$

$$M \geq 2 * \sum_{j \in \mathcal{J}} p_{11j} + \sum_{j \in \mathcal{J}} \max_{i \in \mathcal{I}^{(2)}} p_{2ij}$$

Computational Study
MIP Model vs. LBBD vs. BC

Experiment Setup

- $n = \{10, 20, 50, 100\}$ jobs, $m = \{2, 5, 10\}$ parallel machines
  - Data generated from uniform distribution with different ranges $p_{kij} \sim U[1, 5], U[1, 100]$
  - 100 instances for each combination (except 10 jobs and 10 machines). That is, 2200 instances in total.

- Solved with MIP model, LBBD and BC
  - Limit of 20 min runtime
  - All algorithm tuning features were applied to MIP, LBBD and BC
  - Intel Core i5 2.53 GHz CPU with 4 GB of main memory
  - IBM ILOG CPLEX Optimization Studio version *12.6.2*

Instance with $p_{kij} \sim U[1,5]$

Different classes of FFSP test instances		# unsol. (#unsol. MP)*			Comp. time (s) + 95% C.I.		
n, jobs	(1,RM)	MIP	LBBD	BC	MIP	LBBD	BC
10	(1,2)	0	**0(0)**	**0**	0.35	0.25	14.24
	(1,5)	0	**0(0)**	**0**	0.33	**0.11**	0.18
20	(1,2)	3	1(0)	**0**	51.33	**13.46**	99.89
	(1,5)	0	**0(0)**	**0**	6.27	**0.32**	0.88
	(1,10)	0	**0(0)**	**0**	9.49	**0.35**	0.63
50	(1,2)	31	1(0)	**0**	500.01	**29.28**	204.13
	(1,5)	9	**0(0)**	**0**	374.58	**9.72**	52.58
	(1,10)	1	**0(0)**	**0**	190.60	**7.98**	36.44
100	(1,2)	80	1(1)	**0**	1003.4	**122.41**	486.16
	(1,5)	69	**1(0)**	3(3)	954.01	**137.96**	203.11
	(1,10)	48	1(0)	**0**	961.15	**152.30**	208.23

*Unsol. ins. - instance that an optimal solution can not be found /proven within the limit of 20 min runtime
*Unsol. MP - instance **whose master problem can not be solved** within the limit of 20 min runtime

Instance with $p_{kij} \sim U[1, 100]$

Different classes of FFSP test instances		# unsol. (#unsol. MP)			Comp. time (s) + 95% C.I.		
n, jobs	(1,RM)	MIP	LBBD	BC	MIP	LBBD	BC
10	(1,2)	**0**	2(0)	9	**11.88**	24.65	106.19
	(1,5)	**0**	**0(0)**	1	2.97	**0.44**	16.08
20	(1,2)	32	30(17)	**20**	402.23	**375.05**	732.68
	(1,5)	29	10(10)	**9**	373.09	**200.82**	649.07
	(1,10)	20	11(8)	**5**	298.94	**178.8**	581.26
50	(1,2)	84(1)	**65** (45)	93(1)	1039.3	**801.55**	1127.5
	(1,5)	86	**35** (31)	79	1122.4	**445.91**	1009.8
	(1,10)	73	**15** (12)	61	984.50	**231.48**	812.16
100	(1,2)	97	**79** (62)	89(1)	1178.9	**1000.50**	1104.20
	(1,5)	98	**49** (45)	83	1189.50	**766.59**	1044.60
	(1,10)	95	**34** (32)	67	1177.30	**593.57**	937.77

*Unsol. ins. - instance that an optimal solution can not be found /proven within the limit of 20 min runtime
*Unsol. MP - instance **whose master problem can not be solved** within the limit of 20 min runtime

Optimality Gap of Instance with $p_{kij} \sim U[1, 100]$

Different classes of FFSP test instances		MIP		LBBD		BC	
n, jobs	(1,RM)	# fea. sol	Ave. gap(%)	# fea. sol	Ave. gap(%)	# fea. sol	Ave. gap(%)
10	(1,2)	0	NaN	2	1.03%	9	1.39%
	(1,5)	0	NaN	0	NaN	1	0.46%
20	(1,2)	32	**0.49** %	13	3.29%	20	1.86%
	(1,5)	29	**0.21** %	0	NaN	9	0.79%
	(1,10)	20	**0.20** %	3	0.85%	5	0.46%
50	(1,2)	83	**0.53** %	20	5.78%	92	0.59%
	(1,5)	86	**0.25** %	4	1.04%	79	0.68%
	(1,10)	73	**0.10** %	3	1.27%	61	0.22%
100	(1,2)	97	1.70%	17	2.58%	88	**0.41** %
	(1,5)	98	0.82%	4	0.57%	83	**0.15** %
	(1,10)	95	0.44%	2	0.39%	67	**0.14** %

*fea. sol. - instances that feasible solutions were found, but not proven to be optimal

Conclusion

- We studied scheduling problem of $FF2|(1, RM)|C_{max}$
- We developed the best-known MIP model from literature
- We developed two decomposition-based algorithms (LBBD and BC)
  1. Both of the LBBD and BC algorithms **outperform** the best-known MIP model
  2. LBBD has the best performance in computational time, but suffers from an *issue of unsolved master problems*.
  3. To the best of our knowledge, this is the first study of the implementation of decomposition-based algorithms for solving FFSP.

Future work:

1. Predict which algorithm to use, LBBD or BC?
   Statistic analysis or Machine Learning?
2. Generalization to $FF2|(RM, RM)|C_{max}$

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Thank you